MATHEMATICAL PROGRAMMING AND PRODUCTION: A CASE STUDY OF PORT HARCOURT REFINING COMPANY

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Abstract

The Port Harcourt Refining Company (PHRC) is a subsidiary of the Nigerian National Petroleum Corporation (NNPC). PHRC major problem is the allocation of raw materials for the production of petroleum products. Most persons erroneously believe that the only raw material for the production of some petroleum products like the premium motor spirit (PMS), commonly known as fuel in Local palace, Automotive gas oil (AGO) diesel, kerosene, cooking gas (Methane) etc is the petroleum, but the actual fact is that several chemicals called additives are usually mixed in certain proportion to petroleum at various temperature to get these petroleum products. However, the inability of Port Harcourt Refining Company (PHRC) management to get the profitable proportion of this production mix is responsible for the unbearable scarcity of these petroleum products despite the many turn-around maintenances (TAM) embarked upon by successive government. To solve this production mix problem, the linear programming method, as a mathematical programming tool, which has been acceptable as the best method of solving a production mix problem was applied and the result, which is a feasible solution that will enable the management of the Port Harcourt refining company (PHRC) solve the production mix problem was clearly stated.

Introduction

Low productivity has been the problem of most production companies in Nigeria. This problem has been a source of debate in many quarters and many practical solutions profound but it remains unabated.

The Federal Government has shown greater concern by inaugurating expert committees to look into this problem but to no avail. One of those committees recommended the use of quantitative technique in the production planning, emphasizing on mathematical (linear programming) programming as the best tool of achieving optimal production plan. As stated by the report of the committee, “growth and development strategies should be geared toward stepping up domestic production through the use of viable production mix method”.

However, Port Harcourt Refining Company is also affected by the low productivity problem that has in turn affected various companies that depend on it for production. Production planning according to expert is a function of management concerns with organizing the physical means to be used by an enterprise to manufacture the goods and services, which it provides. Materials, machines and labour have to be blended to enable the production system operate in a cost-effective way. Port Harcourt Refining Company (PHRC) has failed in this task despite the many turn Around-maintenances (TAM) being done to achieve greater productivity level.

In order to attain rationality in production planning, Andrew and Herbert (1983) recommended linear programming as the most effective mathematical programming tool and according to Bassey (2002), non-implementation of this technique is responsible for the low productivity of many enterprises in Nigeria. Also, Taha (2001) stated that Linear programming, as a basic tool for solving a production mix problem, is the most prominent operations research technique designed with strict linear objective function and constraints and origin of linear programming can be traced to a Portland born American mathematicians, George Bermard Dantizig, who accomplished the standard linear programming model in 1947. Dantizing explained that the practical aim of linear programming model developed is to evolve the ability of using simultaneous linear equation to adequately describe a real life problem". He succeeded in using the linear programming model to solve most real life problems. However, Okon (1987) explained that linear programming brought the united airline profits to 564 million dollar with revenue 6.2 billion dollars which is about 6 percent over the 1983 returns while cost grew by less than 2 percent.

Statement of Problem

One of the major problems of manufacturing firm is low productivity and this is caused by the inability of not using the best tool in production decision problem. PHRC does not employ any quantitative method let alone the linear programming in its production planning. It is a well-known fact that PHRC operates below acceptable productivity level which has in turn caused the importation of petroleum products for local consumption. Research work into the production planning implementation of PHRC shows that this firm lacks the skill in determining the best production mix that optimize its objectives. Also the yearly production and profit target of PHRC have never been attained (NNPC Annual Report 2006). From the foregoing, this research was designed to demonstrate to the management of PHRC, the benefits derivable from the application of mathmatical programming tool like the linear programming in its production planning.

Objectives of the Research

1. To ensure, through careful analysis and presentation of facts, that firms in Nigeria adopt linear programming as the best effective tool for production planning.
2. To construct a linear programming model that will enable PHRC realizes her production target.

3. To determine the production mix of
i. Premium motor spirit (PMS).
ii. Automotive gas oil (AGO).
iii. Dual Purpose kerosene
iv. "Fuel oil"
v. Intermediate products

That brings about the attainment of production target and maximizes profit.

4. To demonstrate to the management of PHRC the effectiveness of simplex method in solving L.P model developed for optimal mix.

Scope of the Research

This research is on the production planning of the PORT HARCOURT REFINING COMPANY, ALESA, ELEME IN RIVERS STATE NIGERIA.

The research concentrates on the development of linear programming model for the production planning of the firm. It also deals on the production set-up of the firm, which included the raw material, machine-time, manpower and their availability per month.

Limitations of the Research

The limitations of this research are in two folds. First, the techniques of the production mix (L.P) has its own limitations and secondly, because of the highly politicizing of PHRC inputs and outputs, actual facts are difficult to come by. From linear programming problem, solution variable can have any value but in some cases, some of the decision variables can have integer values only. Also the linear programming model does not take into account the effect of time and some time both objectives function and constraints cannot be expressed in linear form. Also,

a. It is not easy to define what a specific objective function is.
b. Relationship between input and output in real life are not always linear but for linear programming all functions must be linear.
c. In real life, there is diminishing, and increasing in production, yet this method assumed constant increase in returns.

Amidst these limitations, linear programming still proves to be useful operatoin for solving mix problems.

Data Collection and Presentation

The data collected are secondary data. They were got from direct observation of the company's annual and monthly statistics bulletin. The data sought for in this research work are:

1. The main products of PHRC; which are
i. Premium motor spirit (PMS)
ii. Automotive Gas oil (AGO)
iii. Fuel oil
iv. Intermediate products or feed stock
2. The profit of a unit of each of the company's product
3. The maximum demand of each product by consumers over a period of time, which is one month in this research.

4. The raw materials used in the production of a unit of each product and their maximum availability.
5. The machines used in the production, the maximum capacities of each machine and the production time per unit of each product.

Raw Materials

S/NO	Raw Materials	PMS	AGO	DPK	Fuel oil	Interpdt	
1.	Hydrogen gas	0.02	0.03	0.07	0.06	0.08	11.257
2.	Water (H_2O)	0.01	0.05	0.06	0.09	0.010	Infinity
3.	Propylene dichloride	0.56	0.16	0.18	0.21	0.26	13.245
4.	NaoH	0.21	0.21	0.31	0.34	0.31	26.722
5.	Tetraethyl-lead	0.06	0.04	0.06	0.05	0.04	12.678
6.	Con. Sulphuric Acid (H_2S0_4)	0.028	0.43	0.43	0.04	0.00	10.500
7.	Petroleum (crude oil)	0.26	0.08	0.16	0.21	0.21	157.436

Unit of measurement is metric ton

Profit Per Unit of Product

PRODUCT	PMS	AGO	DPK	PUEL	INTER PDT
Profit N	25.75	28.79	18.65	0.65	7.85

Machine Requirements

S/No	Machin e	Time to Produce a Unity of					Max. time liability per month
		Pms	Ago	Dpk	Duel	Inter pdit	
1.	Pump	0.42	.039	0.88	0.6	0.30	Infinity
2.	Boiler	1.02	0.96	0.67	0.98	0.64	72hrs
3.	Mining	0.40	0.67	0.60	0.50	0.20	72hrs

Unit of measurement here is hour

Table 4 consumers' demand

S/NO	PRODUCT	DEMAND (LITERS)
1.	PMS	651.531,666.7
2.	AGO	159,898,000
3.	DPK	65,879,000
4.	Fuel Oil	3.413,333
5.	Intermediate pdts	6,377,000

Model Specification

Linear programming model specification for PHRC Port Harcourt Refining Company (PHRC) produces many finished products but the research will consider the five main products with three grades of imitation, raw materials, demand and machine time.

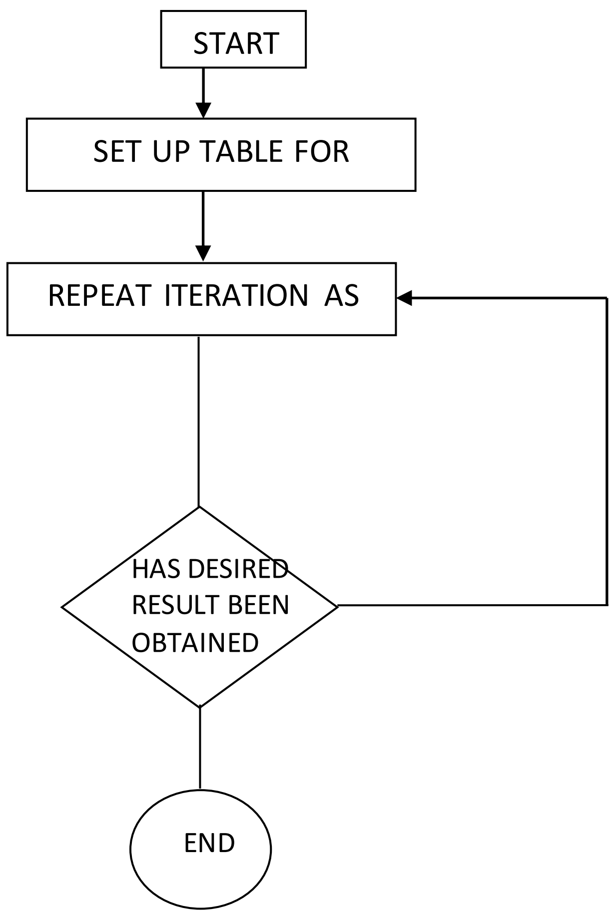
The L.P model is thus

Max Z = $C_1 X_1 + C_2 X_2 + C_1 X_3 + C_4 X_4 + C_5 X_5$

s.t
$a_{11}x_1 + a_{11}x_2 + a_{13}x_3 + a_{14}x_4 + a_{25}x_5 \le b_1$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 \le b_2$
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{24}x_4 + a_{35}x_5 \le b_3$
$a_{111}x_1 + a_{112}x_2 + a_{113}x_3 + a_{114}x_4 + a_{115}x_5 \le b_{11}$
$x_1, x_2, x_3, x_4, x_5 \le 0.\ bi \le 0.$

The standard linear programming model
Max Z = $\sum C_i X_i$
s. t
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 \le b_1$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 \le b_2$
$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + a_{i4}x_4 + a_{i5}x_5 \le b_{11}$
$x_i \ge 0$, bi ≥ 0

The Structure of the Algorithm



Data Analysis

For the data analysis of PHRC production mix problem, I will consider it in two parts

i. Decision variable and
ii. Objective functions parts

Decision Variable Part

PHRC produces many products but I will consider major five of her products. These include:

a. Premium motor spirit (PMS)
b. Dual purpose kerosene (DPK)
c. Automotive Gas oil (AGO)
d. Fuel oil
e. Intermediate products. (Feed stock).

The number of units. (litres) to be produced for the five considered products are X_1, X_2, X_3, X_4, X_5 respectively.

Objective Function

Litres of PMS, DPK, AGO, Fuel oil and intermediate products return unit profit of N25.75, N28, 79, N18.65 N o.65 and N7.85 respectively and their total profit contribution are 25.75 X_1, 28.79 X_2, 18.65 X_3 0.65 X_4 and 7.85 X_5 respectively

L.P Model

$\text{Max } Z = 25.75 x_1 + 28.79 x_2 + 18.65 x_3 + 0.65 x_4 + 7.85 x_5$

s.t

$0.02x_1 + 0.03 x_2 + 0.07 x_3 + 0.06 x_4 + 0.08 x_5 \leq 11.267$

$0.26x_1 + 0.16 x_2 + 0.18 x_4 + 0.08 x_5 \leq 16,322$

$0.21x_1 + 021x_2 + 0.31 x_3 + 0.34 x_4 + 0.04 x_5 \leq 12,678$

$0.06x_1 + 0.04x_2 + 0.06 x_3 + 0.05x_4 + 0.04 x_5 \leq 12,678$

$0.28x_1 + 0.43x_2 + 0.43 x_3 + 0.04x_4 + 0.00 x_5 \leq 10,500$

$0.16x_1 + 0.08x_2 + 0.16 x_3 + 0.21x_4 + 0.21 x_5 \leq 157, 436$

$X_1 , x_2, x_3 , x_4 , x_5 \geq 0$

Model Solution

The model solution contains some parts of the computer output of the linear programming model of PHRC.

Maximum objective function value

N122, 912, 754, 036.61k

VARIABLE VALUE

X_1	650,530,657.8
X_2	53,898.004
	5,879.004
X_3	3,412.400
	6,370.010
X_4	
X_5	

Model Validation/Testing

Model validation is a process of trying in verify whether the model as formulated, predicts adequately the behaviour of the system under study. The generally acceptable method of checking the validity of a model is to compare is output with historical data. The model is valid, according to Taha (2001) if it produces past performance. From Taha's statement on validity, it can be seen that the monthly demand for PMS which is 651, 531,666.7 National wide deviates slightly with a very small percentage. So also was the demand of AGO, DPK fuel oil and intermediate products.

Optimum Solution Summary

This had three parts, the first which displays the number of interaction (6) that gave the maximum, objective function of N22,912,754,036.61k. The second part gives the maximum values of variables, namely, quantity of PMS (x_1) is 650, 530, 567.8 liters, AGO (x_2) is 153,8987,004, DPK (x_3) is 65,879, 004 fuel oil (x_4) is 153,898 004 and intermediate products (x_5) is 6,370,010.

Also shown are the relative profit coefficients of the products in the objective function and their contributions.

VARIABLE	VALUE	OBJECTIVE COEFF.	OBJ. VALUE COEFF.
X_1	650,530,657.8	25.75	16751164438.35
X_2	153,898,004	28.79	4430723535.16
X_3	5,879,004	18.65	1228643424.60
X_4	3,412,400	0.65	2218060.00
X_5	6,370.010	7.85	50004578.50

CONSTRAINTS	**PHS**	**SLACK(-)/SURPLUS (X)**

1<	1126	266
2<	1345	0.000
3<	16722	45.6
4<	12678	0.26
5<	10500	512
6<	157436.67	1,256
7<	651,531,666.7	625.4
8<	159.898,000	0.000
9<	65,879,000	423
10<	3,413,333	280
11<	6.377,000	0.000
12<	72,22	30.2
13<	72.00	13.4

Company. The last part of this section accounts for the current right-hand side value of the constraint and their slacks (-). There are no surpluses (X) since all constraints are of the type ≤. These slacks are the amounts by which the constraint are over-satisfied.

Also, the constraints 6-11 are representing the demand constraint and constraints 12-13 the machine constraint.

Discovering

The research enable me to discover that, despite the many excuses usually given by port Harcourt refinery company (PHRC) for scarcity of major cause of scarcity is the non implementation of qualitative technique in her production process. This research produces a better cost effective production policy for PHARC which can generate, on a monthly basis N22,912,754,03. 81k with the production of 650,530 657.8 liters, 153,898,004 liters, 65,879004 liters, 312400 liters and 6,370,010 liters of PMS, DPK, AGO, Fuel oil and intermediate products respectively.

Furthermore, the excess in demand of litres of some of the main products proved the existence of scarcity and capacity underutilization. Greater profit can be enhanced if excess raw material can be channel to these areas so as to maximize PHRC profit. Finally, from the out-put of the model, there are approximately 30.53 hrs in which the machine lie idle in a month. This also points to the capacity under unitization of the company. This should not be accommondated, instead the machine should be leased out to joint venture operators (JVO) like shell Agip, Cheveron, Texaco etc.

Recommendation

From the discoveries, the research found out that some raw materials are almost used up in the production process and therefore suggest that the excess be reduced and the money channeled to the acquisition of the more needed ones.

Furthermore, the research recommend that PHARC should as matters of urgency apply this quantitative technique in the production process so as to avert the dreadful monster called fuel scarcity that has plagued this nation.

Conclusion

In conclusion, the production mix problem under consideration has been adequately solved and it is believed that management of PHRC will enforce this technique so effectively demonstrated in her production problem.

It is also believed that the cry for some of the finished productions and be well tackled by looking at the excess demand and the used raw material and blending the process for more cost effective way.

References

Andrew, V. Herbert, F.S (1984). Quantitative Analysis for Business. New Jersey, U.S.A Prentice – Hall Ine.

Bassesy, J.E (2002) or- Notes Unpublished Lecture Notes London Imperial College.

Bernd Gartner, Jiri Matousek (2006). Understanding and Using Linear Programming, Berhn Springer.

Chukwu, W.I.E (2004). Operations Research I Unpublished Lecture Nsukka: Department of Statistics, university of Nigeria.

Chigbu, P.E (2004). Statistics for National Development: the Rationale for 30th Anniversary Celebrations. Department of Statistic University of Nigeria.

Jalaludin Adbullah, Optimization by the Fixed Point Method, Version 1.97, (august, 2007).

Jonah, V.A (2003). Linear Programming in Managerial Decision Marking. Unpublished B.Sc thesis Nsukka: Department of Statistics/Economics. University of Nigeria.

Taha, I.T A, (2006). Operation Research, an Introduction, 8th Edition Prentice hall.

Taha, HA. (2001). Operation Research an Introduction: New Jersey, U.S.A Prentices- Hall Inc.

Turban, E. (1993). Decision Support and Expert Systems; Management Support Systems. New York Macmillan Publishing